

Electroexcitation of the Roper resonance in the relativistic quark models

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The amplitudes of the transition $\gamma^*N \rightarrow P_{11}(1440)$ are calculated within light-front relativistic quark model assuming that the $P_{11}(1440)$ is the first radial excitation of the $3q$ nucleon state. The results are compared with those obtained in close approaches by other authors and with standard nonrelativistic calculations. One of the reasons for this study was to present all these results within unified definition of helicity amplitudes consistent with the definition used in the extraction of the helicity amplitudes from experimental data in one-pion electroproduction. The results of relativistic quark models are qualitatively in good agreement with each other and differ strongly from nonrelativistic calculations. At small Q^2 , these results for the transverse amplitude $A_{1/2}$ are consistent, but fail to reproduce experimental data. The most probable explanation of this discrepancy is the absence of pion cloud contribution in the approaches under consideration.

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I. INTRODUCTION

The amplitudes of the electroexcitation of the Roper resonance on proton are expected to be obtained from the CLAS π^+ electroproduction data at $1.7 < Q^2 < 4.2 \text{ GeV}^2$. There are already amplitudes [1] extracted from the preliminary data [2, 3], and the final results will be available soon. The results at $1.7 < Q^2 < 4.2 \text{ GeV}^2$ combined with the previous CLAS data at $Q^2 = 0.4, 0.65 \text{ GeV}^2$ [4, 5, 6] and with the information at $Q^2 = 0$ [7] will give us knowledge of the Q^2 evolution of the $P_{11}(1440)$ electroexcitation in wide Q^2 region. This information can be very important for understanding of the nature of the Roper resonance which has been the subject of discussions since its discovery [8], because the simplest and most natural assumption that this is a first radial excitation of the $3q$ nucleon state led to the difficulties in the description of the mass of the resonance.

In order to utilize the information on the the electroexcitation of the Roper resonance, it is important to have an understanding of which are the quark model predictions for the Q^2 evolution of the transition $\gamma^*N \rightarrow P_{11}(1440)$. It is known that with increasing Q^2 , when the momentum transfer becomes larger than the masses of the constituent quarks, a relativistic treatment of the electromagnetic excitations, which is important already at $Q^2 = 0$, becomes crucial. The consistent way to realize the relativistic treatment of the $\gamma^*N \rightarrow N^*$ transitions is to consider them in the light-front (LF) dynamics [9, 10, 11]. Within this framework, one can set up an impulse approximation and avoid difficulties

caused by different momenta of initial and final hadrons in the $\gamma^*N \rightarrow N^*$ transition. For the $P_{11}(1440)$ such approach has been realized in Refs. [12, 13, 14]. However, quantitatively the predicted results differ from each other and there are inconsistencies in the signs of the amplitudes obtained in Refs. [12, 13, 14]. For this reason, we found it important to perform calculations of the $\gamma^*N \rightarrow P_{11}(1440)$ amplitudes in the LF relativistic quark model and to find sources of disagreement of the results obtained in Refs. [12, 13, 14]. We will also specify and compare different definitions of the $\gamma^*N \rightarrow P_{11}(1440)$ helicity amplitudes in order to eliminate inconsistencies caused by the differences of definition.

II. DEFINITIONS OF THE $\gamma^*N \rightarrow P_{11}(1440)$ HELICITY AMPLITUDES

The $\gamma^*p \rightarrow P_{11}(1440)$ helicity amplitudes extracted in Refs. [1, 2, 3] from electroproduction data are defined through the $P_{11}(1440)$ contribution to the multipole amplitudes of the reaction $\gamma^*p \rightarrow \pi N$: M_{1-}^R , S_{1-}^R . The commonly used formulas presented in Refs. [15, 16] were used:

$$A_{1/2} = a \text{Im} M_{1-}^R(W = M), \quad (1)$$

$$S_{1/2} = -\frac{a}{\sqrt{2}} \text{Im} S_{1-}^R(W = M), \quad (2)$$

$$a = \sqrt{2\pi \frac{q_\pi}{K} \frac{M}{m} \frac{\Gamma}{\beta_{\pi N}}} / C, \quad (3)$$

$$C = -\sqrt{\frac{1}{3}} \quad \text{for } \gamma^*p \rightarrow p\pi^0, \quad (4)$$

$$C = -\sqrt{\frac{2}{3}} \quad \text{for } \gamma^*p \rightarrow n\pi^+, \quad (5)$$

where Γ and M are the total width and mass of the resonance, $\beta_{\pi N}$ is its branching ratio to πN channel, m is

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the mass of the nucleon, $K = (M^2 - m^2)/2M$, q_π is the center of mass momentum of the pion at the resonance position.

The helicity amplitudes defined through Eqs. (1-5) include the relative sign between $g_{NN\pi}$ and $g_{RN\pi}$ coupling constants which determines the relative contributions of the Born term and the $P_{11}(1440)$ to the reaction $\gamma^*N \rightarrow \pi N$.

Let us define the $\gamma^*N \rightarrow N^*$ transition current matrix element in terms of the form factors $F_1^*(Q^2)$ and $F_2^*(Q^2)$:

$$\langle N^* | J_\mu | N \rangle = e \bar{u}(P^*) \Gamma_\mu u(P), \quad (6)$$

$$\Gamma_\mu = (k^2 \gamma_\mu - (k\gamma)k_\mu) F_1^*(Q^2) + i \sigma_{\mu\nu} k^\nu F_2^*(Q^2). \quad (7)$$

Using the definitions (1-5) one can express the helicity amplitudes $A_{1/2}$, $S_{1/2}$ through the form factors $F_{1p}^*(Q^2)$ and $F_{2p}^*(Q^2)$ of the $\gamma^*p \rightarrow P_{11}(1440)$ transition:

$$A_{1/2} = -c [Q^2 F_{1p}^*(Q^2) - (M + m) F_{2p}^*(Q^2)], \quad (8)$$

$$S_{1/2} = -c \frac{k_{cms}^*}{\sqrt{2}} [F_{2p}^*(Q^2) + (M + m) F_{1p}^*(Q^2)], \quad (9)$$

$$c = -\xi_R \sqrt{\frac{M k_{cms}^{*2}}{mK} \frac{4\pi\alpha}{(M+m)^2 + Q^2}}, \quad (10)$$

where ξ_R is the relative sign between $g_{NN\pi}$ and $g_{RN\pi}$ coupling constants, and $\alpha = e^2/4\pi = 1/137$.

In quark model calculations another definition of the $\gamma^*N \rightarrow P_{11}(1440)$ helicity amplitudes through the $\gamma^*N \rightarrow N^*$ transition current matrix elements commonly is used [12, 17]:

$$A_{1/2}^q = b \langle N^{*+}, S_z^* = \frac{1}{2} | J_\epsilon | p, S_z = -\frac{1}{2} \rangle, \quad (11)$$

$$S_{1/2}^q = b \frac{k_{cms}^*}{Q} \langle N^{*+}, S_z^* = \frac{1}{2} | J_\epsilon | p, S_z = \frac{1}{2} \rangle, \quad (12)$$

$$b = \left[\frac{2\pi\alpha}{K} \right]^{1/2}. \quad (13)$$

Here it is supposed that the virtual photon moves along the z-axis in the N^* rest frame and its 3-momentum is k_{cms}^* , $P^* = P + k$, $Q^2 \equiv -k^2$.

In order to compare the quark model predictions obtained using Eqs. (11-13) with the results extracted from experimental data via Eqs. (1-5), the amplitudes $A_{1/2}^q$, $S_{1/2}^q$ should be multiplied by $-\xi_R$: $A_{1/2} = -\xi_R A_{1/2}^q$, $S_{1/2} = -\xi_R S_{1/2}^q$.

In some papers (for example, in Ref. [12]), the N , N^* helicities are used in formulas (11,12) instead of the spin projections. This leads to the opposite sign for the amplitude $A_{1/2}^q$.

III. THE $\gamma^*p \rightarrow P_{11}(1440)$ AMPLITUDES IN THE RELATIVISTIC QUARK MODEL

The calculations of the $\gamma^*N \rightarrow P_{11}(1440)$ amplitudes we have performed in the relativistic quark model of Ref. [18], constructed for radiative transitions of

hadrons in the infinite momentum frame, where the initial hadron moves along the z-axis with momentum $P \rightarrow \infty$, and the momentum of the photon is $k = (\mathbf{k}_\perp, -\frac{M^2 - m^2 - \mathbf{k}_\perp^2}{4P}, \frac{M^2 - m^2 - \mathbf{k}_\perp^2}{4P})$. Such approach is analogous to the LF calculations [9, 10, 11, 12, 13, 14]. The formulas for the transition $\gamma^*N(\frac{1}{2}^+) \rightarrow N(\frac{1}{2}^+)$ are presented in detail in Ref. [19], where the model of Ref. [18] was used to describe the Q^2 evolution of the $\gamma^*N \rightarrow N$ form factors. These formulas can be directly used for the evaluation of the $\gamma^*p \rightarrow P_{11}(1440)$ amplitudes if we will replace the radial part of the final state nucleon wave function by that of the $P_{11}(1440)$: $\Phi_N(M_0'^2) \rightarrow \Phi_R(M_0'^2)$, where M_0' is the invariant mass of quarks in the final state hadron. Under the assumption that $P_{11}(1440)$ is a radial excitation of the 3-quark nucleon state, we have

$$\Phi_R(M_0'^2) = N(\beta^2 - M_0'^2) \Phi_N(M_0'^2). \quad (14)$$

The parameters N and β are determined by the conditions:

$$\int \Phi_R(M_0'^2) \Phi_N(M_0'^2) d\Gamma = 0, \quad \int \Phi_R^2(M_0'^2) d\Gamma = 1. \quad (15)$$

Here $d\Gamma$ is the phase space volume, $M_0'^2 = \sum_i \frac{\mathbf{k}_{i\perp}^2 + m_q^2}{x_i}$, quark momenta are parametrized by $\mathbf{k}_i = x_i \mathbf{P} + \mathbf{k}_{i\perp}$, $i = 1, 2, 3$ denotes the quark, and m_q is the quark mass.

As in Refs. [18, 19], we have taken the radial part of the nucleon wave function in the Gaussian form: $\Phi(M_0'^2) \sim \exp(-M_0'^2/6\alpha_{HO}^2)$. The only parameters of the approach are the quark mass ($m_q = 0.22 \text{ GeV}$) and the harmonic-oscillator parameter ($\alpha_{HO} = 0.38 \text{ GeV}$) which were found in Ref. [18] from the description of the static properties of the nucleon.

We have found the relative sign between $NN\pi$ and $P_{11}(1440)N\pi$ amplitudes (ξ_R) by relating these amplitudes to the matrix elements of the axial-vector current ($\bar{u}\gamma_\mu\gamma_5 u$) using PCAC. The formulas for these matrix elements were taken from Refs. [18, 19] (see also Ref. [20]). With the definition (14) for the radial part of the $P_{11}(1440)$ wave function, we have obtained $\xi_R = 1$. The same sign was obtained in Ref. [13] using 3P_0 model of Ref. [21].

Our results for the $\gamma^*p \rightarrow P_{11}(1440)$ helicity amplitudes are presented in Fig. 1 along with the results obtained in Refs. [12, 13, 14]. In accordance with the discussion in Sec. II, we took into account difference of the signs of the amplitude $A_{1/2}^q$ obtained via Eq. (11) and using the corresponding formula from Ref. [12]. We have corrected inaccuracies in Eqs. (18) of Ref. [12] and recalculated the helicity amplitudes using the wave functions from that work. In Ref. [12], we did not find information on the sign ξ_R . The results from Ref. [12] presented in Fig. 1 correspond to the sign obtained in this work and in Ref. [13].

The quark mass $m_q = 0.22 \text{ GeV}$ coincides with that from Refs. [13, 14] and agrees with the light-quark mass obtained from the description of baryon masses in Ref. [22]. The wave functions in Refs. [12, 13] are taken also

in the Gaussian form. Our harmonic-oscillator parameter $\alpha_{HO} = 0.38 \text{ GeV}$ is close to that from Ref. [13]. The parameters used in Ref. [12] are different; by this reason above 1 GeV^2 , the slope of the amplitudes corresponding to Ref. [12] is larger.

The wave functions in Ref. [14] (see also Ref. [23]) differ from simple harmonic-oscillator wave functions due to the effects connected with the one-gluon-exchange interaction which is taken into account in the form used in Ref. [22]. In Refs. [14, 23], the effects connected with the form factors and anomalous magnetic moments of quarks are also taken into account. As it is shown in Ref. [23], these effects lead to the suppression of the absolute values of the amplitudes $A_{1/2}$ and $S_{1/2}$.

In order to demonstrate how important is the role of the relativistic effects, in Fig. 1 we have presented the results obtained according to the standard nonrelativistic calculations [24, 25] using the definitions (11,12). Our parameters and the sign ξ_R obtained in this work have been used. Let us note that the sign of our nonrelativistic amplitudes differs from the sign of the amplitudes in Refs. [24, 25]. In Refs. [24, 25], the sign ξ_R was found not within the quark model, but was fixed using experimental data on the amplitude $A_{1/2}$ at $Q^2 = 0$. Our sign of the nonrelativistic amplitudes is in agreement with that obtained in Ref. [13].

From Fig. 1 it is seen that the relativistic effects strongly change the quark model predictions for the $\gamma^*p \rightarrow P_{11}(1440)$ transition. The results obtained in the relativistic approaches show nontrivial behavior of the transverse amplitude $A_{1/2}$, which changes sign at $Q^2 \simeq 0.1 \text{ GeV}^2$. Such behaviour of this amplitude is caused by the fact that due to the correct treatment of the relativistic effects, the form factor $F_2^*(Q^2)$ changes the sign at $Q^2 \simeq 0.15 \text{ GeV}^2$ from positive at $Q^2 < 0.15 \text{ GeV}^2$ to negative. In the nonrelativistic calculations this form factor is negative at all Q^2 . The form factor $F_1^*(Q^2)$ is positive at all Q^2 in both relativistic and nonrelativistic approaches.

IV. SUMMARY

In this work we have calculated the $\gamma^*p \rightarrow P_{11}(1440)$ transition amplitudes in the LF relativistic quark model.

The amplitudes are presented along with the results obtained in close approaches in Refs. [12, 13, 14] and with nonrelativistic amplitudes. All results are presented within unified definition of helicity amplitudes consistent with the definition used in the extraction of the helicity amplitudes from experimental data in one-pion electroproduction. The results of relativistic quark models are qualitatively in good agreement with each other showing nontrivial behaviour of the transverse $A_{1/2}$ amplitude. Being negative at $Q^2 = 0$, this amplitude changes sign at $Q^2 \simeq 0.1 \text{ GeV}^2$ and becomes large and positive at $Q^2 \simeq 1 \text{ GeV}^2$; then it slowly falls with increasing Q^2 . The longitudinal amplitude $S_{1/2}$ is large and positive at $Q^2 = 0$; with increasing Q^2 it decreases.

At $Q^2 < 0.6 \text{ GeV}^2$, the results of different relativistic quark models for the transverse amplitude $A_{1/2}$ are consistent, but significantly higher than experimental data. From our point of view, this is an indication on the large pion cloud contribution to the $\gamma^*N \rightarrow P_{11}(1440)$ transition which is expected to be significant at small Q^2 . When final results on the helicity amplitudes at $1.7 < Q^2 < 4.2 \text{ GeV}^2$ will become available, complete simultaneous description of the nucleon form factors and the $\gamma^*N \rightarrow P_{11}(1440)$ amplitudes will be necessary. Such investigation will be important to find and specify the size of the pion cloud contribution, to investigate the possible mixing between N and $P_{11}(1440)$, as well to understand the role of other effects such as the quark form factors and anomalous magnetic moments, and probably the quark mass dependence on the quark virtualities and therefore on Q^2 .

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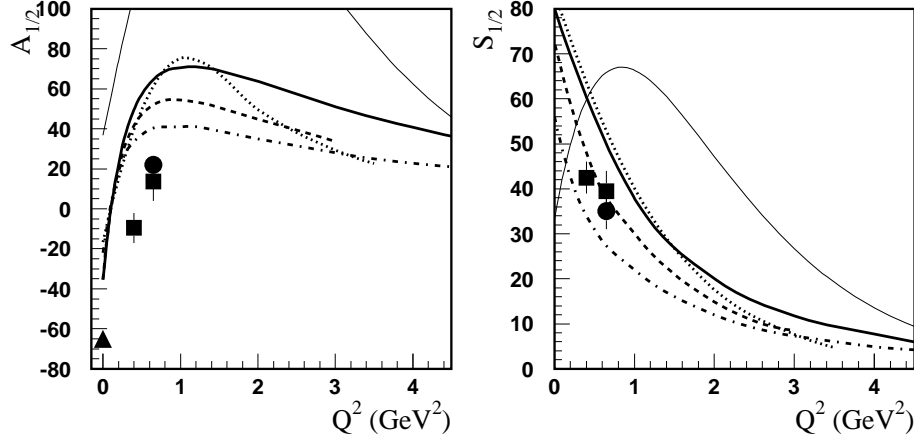


FIG. 1: Helicity amplitudes for the $\gamma^* p \rightarrow P_{11}(1440)$ transition (in $10^{-3} \text{ GeV}^{-1/2}$ units). Full boxes and circles are the results obtained in the analysis of π electroproduction data in Refs. [4, 5]. Full triangle at $Q^2 = 0$ is the PDG estimate [7]. Thick lines correspond to the LF relativistic quark models. Thick solid lines are the results obtained in this paper. Dashed and dash-dotted lines are the results of Refs. [13, 14], respectively. Dotted lines correspond to the results of Ref. [12] recalculated according to the discussion in Sec. III. Thin solid lines are the results of the nonrelativistic calculations [24, 25] obtained using our parameters and the sign ξ_R between $g_{NN\pi}$ and $g_{RN\pi}$ coupling constants found in this work.

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